Dr John Armstrong

Department of Mathematics

Faculty of Natural and Mathematical Sciences

2025.09.09

Dr John Armstrong

Department of Mathematics

Andros25

Collective Pensions in the UK

Collective Pensions in the UK

- Shared-indexation Collective Defined Contribution schemes
- Optimal collective pensions in complete markets

Collective Pensions in the UK

- Shared-indexation Collective Defined Contribution schemes
- Optimal collective pensions in complete markets

Contributors

- Pension Policy Institute, Catherine Donnelly, Cristin Buescu, James Dalby, Rohan Hobbs

Pension Schemes Acts 2015 and 2021

- Defined Benefit (DB)
- Defined Contribution (DC)
- Shared Risk (aka Defined Ambition) (2015)
- Collective Defined Contribution (CDC) (2021) - Royal Mail launched Oct 2024.

Pension Schemes Acts 2015 and 2021

- Defined Benefit (DB)
- Defined Contribution (DC)
- Shared Risk (aka Defined Ambition) (2015)
- Collective Defined Contribution (CDC) (2021) - Royal Mail launched Oct 2024.

Collective benefits

"The Act (2015) includes measures that will enable workplace and personal pension schemes to provide 'collective benefits'. These are provided by allowing schemes to be run in a way that shares risks among members by pooling their assets. This means that when a member retires, they can receive an income from the shared assets of the scheme.

"Evidence from other countries suggests that by sharing the risks among members, schemes providing 'collective benefits' may provide more stable outcomes than individual Defined Contribution schemes currently available in the UK. For example, while members are saving for retirement they can get some degree of protection from fluctuations in the financial markets."

Industry Expectations

Source: Aon: The Case for Collective DC

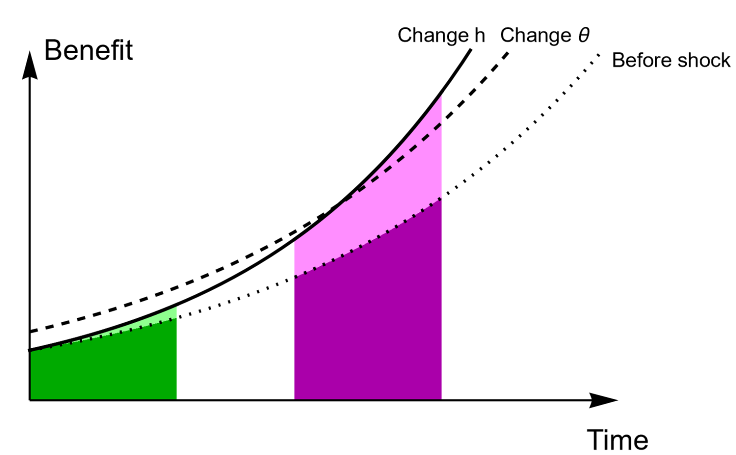
Shared-indexation CDC

- Assume all individuals are identical apart from their generation ξ ($\xi = 0, \ldots, M-1$).
- In exchange for a given contribution C_t^ξ, members accrue a nominal benefit each year B_t^ξ

$$B_t^{\xi,cum} = \theta_t(1+h_t)B_{t-1}^{\xi,cum} + B_t^\xi$$

- In retirement you receive $B_t^{\xi,cum}$, but before retirement it is a purely nominal figure.
- h_t is the *prevailing level of indexation* at time t and is chosen so assets match liabilities.
- The effect of h_t is compounded.
- h_t constrained to lie in some range e.g. $[0, \mathrm{CPI}_t + 5\%]$.
- θ_t is the level of cuts ($\theta_t < 1$) or bonuses ($\theta_t > 1$). In normal years $\theta_t = 1$. So the effect of θ_t is not compounded.
- *Shared-indexation* because everybody receives the same level of indexation.

Stylised diagram of shared-indexation



Matching assets to liabilities

h_t and θ_t are chosen to ensure assets before new payments in and out (A_{t-}) satisfy

$$A_{t-} = \theta_t \sum_{\xi=0}^{M-1} \sum_{\ell=0}^{\infty} (1+h_t)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_{t-1}^{\xi,\mathrm{cum}} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

- $\mathbf{1}_{t+\ell}^{R.\xi}$ - indicator of whether generation ξ is retired in year $t+\ell$
- $p^{\xi}(t, t+\ell)$ - Survival probability from t to $t+\ell$
- $B_{t-1}^{\xi,\mathrm{cum}}$ - Accumulated benefit over last period
- N_t^{ξ} - Number of survivors in generation ξ at time t
- $(1+r)^{-\ell}$ - Discount rate - assuming fixed expected return r for all generations. Note, CDC funds invest in risky assets.
- $(1+h)^{\ell+1}$ - Compounded indexation

Simplified slightly to omit inflation and varying investment strategy with age.

The steady-state equation

We have said h_t and θ_t are chosen to ensure assets before new payments in and out (A_{t-}) satisfy

$$A_{t-} = \theta_t \sum_{\xi=0}^{M-1} \sum_{\ell=0}^{\infty} (1+h_t)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_{t-1}^{\xi,\mathrm{cum}} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

Let A_t be assets after new payments in and out

$$A_t \neq \sum_{\xi=0}^{M-1} \sum_{\ell=1}^{\infty} (1+h_t)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} (B_{t-1}^{\xi,\mathrm{cum}} + B_t^{\xi}) p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

unless this steady-state equation holds

$$\sum_{\xi=0}^{M-1} C_t^{\xi} = \sum_{\xi=0}^{M-1} \sum_{\ell=1}^{\infty} (1+h_t)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_t^{\xi} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

Flat accrual vs Dynamic accrual

$$\sum_{\xi=0}^{M-1} C_t^{\xi} = \sum_{\xi=0}^{M-1} \sum_{\ell=1}^{\infty} (1+h)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_t^{\xi} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

Flat accrual vs Dynamic accrual

$$\sum_{\xi=0}^{M-1} C_t^{\xi} = \sum_{\xi=0}^{M-1} \sum_{\ell=1}^{\infty} (1+h)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_t^{\xi} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

In a **flat-accrual** scheme

- Benefits are proportional to contributions irrespective of generation g and time t.
- "DB-lite'' scheme (Royal Mail)
- The ratio $\frac{B}{C}$ is chosen so the fund is in a steady-state when $h = \text{target}$
- Manifestly unfair pricing, only viable for single-employer
- h will mean revert

Flat accrual vs Dynamic accrual

$$\sum_{\xi=0}^{M-1} C_t^{\xi} = \sum_{\xi=0}^{M-1} \sum_{\ell=1}^{\infty} (1+h)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_t^{\xi} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

In a **flat-accrual** scheme

- Benefits are proportional to contributions irrespective of generation g and time t.
- "DB-lite'' scheme (Royal Mail)
- The ratio $\frac{B}{C}$ is chosen so the fund is in a steady-state when $h = \text{target}$
- Manifestly unfair pricing, only viable for single-employer
- h will mean revert

In a **dynamic-accrual** scheme

- The cost of new benefits is chosen to ensure the steady-state equation holds
- Suitable for multi-employer schemes (2024 DWP Consultation)
- h will walk between barriers

$$C_t^{\xi} = \sum_{\ell=1}^{\infty} (1+h)^{\ell+1} (1+r)^{-\ell} N_t^{\xi} B_t^{\xi} p^{\xi}(t,\ell) \mathbf{1}_{t+\ell}^{R,\xi}.$$

Cross-subsidies in flat-accrual CDC

- Inflation 2.0%
- Risk-free interest rate 4.36%
- Equity returns 7.73%

Cross-subsidies in flat-accrual CDC

- Inflation 2.0%
- Risk-free interest rate 4.36%
- Equity returns 7.73%

- Unfairness in index-linked DB scheme over 40 years $(1+0.0236)^{40} \approx 2.5$
- Unfairness in flat-accrual CDC scheme targeting $h = \mathrm{CPI}$ over 40 years $(1+0.0573)^{40} \approx 9.2$

Long-term impact of cross-subsidies

- All individuals contribute an amount $C(1+g)^t$ to a DB fund each year from the time they join a scheme to retirement n years later.
- Each year the same number of members join
- Each year, their current benefit entitlement grows to match inflation (which is constant and equal to i) and they receive an additional benefit entitlement $B(1+g)^t$.
- Money invested in a riskless asset which grows at a rate r.
- The scheme has an infinite number of investors and there is no systematic longevity risk.
- The regulator insists assets=liabilities at all times.

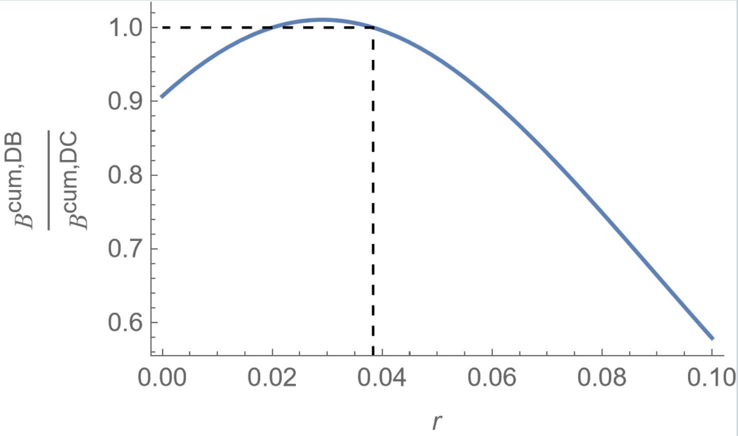
Theorem

The ratio of benefits received from a flat-accrual CDC scheme or DB scheme to the benefits received in a riskless DC + annuity scheme is

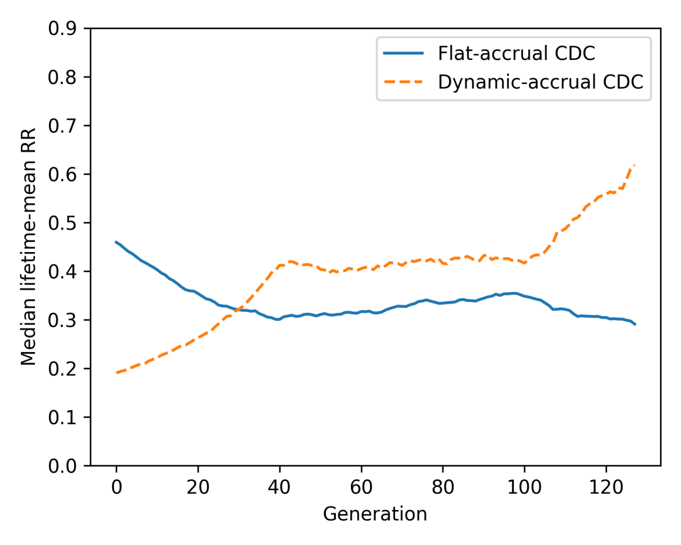
$$\frac{(\alpha-1)(i+1)n\alpha^n(r-g)\left((i+1)^n-(g+1)^n\right)}{(r+1)(i-g)\left(\alpha^n-1\right)\left((r+1)^n-(g+1)^n\right)}.$$

where $\alpha=(1+r)/(1+i)$. As CDC funds invest in risky assets, this should be seen as an estimate for the expected ratio for CDC funds.

Calibrated to OBR Figures

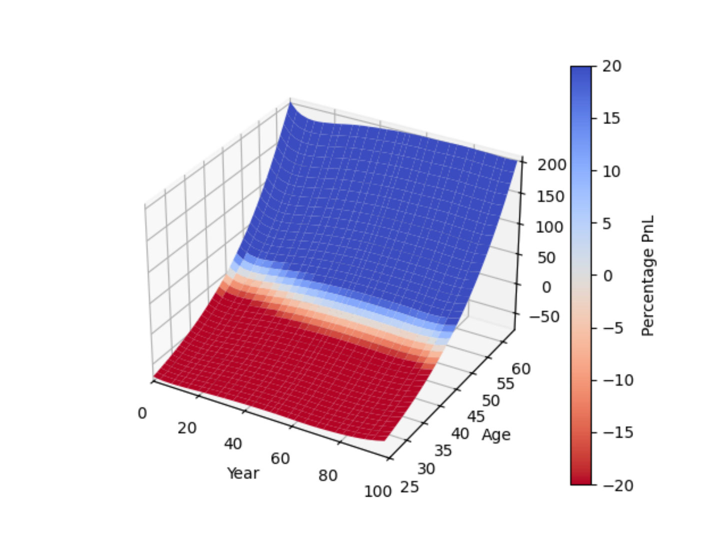


Simulation Results (rich economic scenario generator)

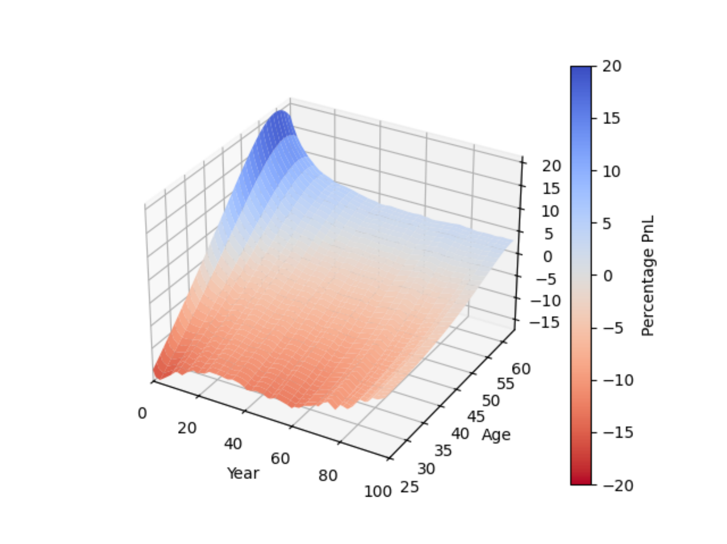


Cross subsidies in a flat-accrual scheme

Black-Scholes value of income (at time 0) vs cost of benefits for fixed-accrual scheme.

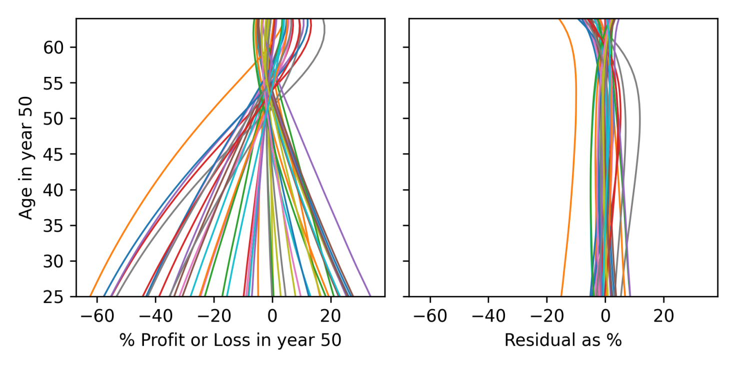


Average cross-subsidies in dynamic accrual CDC



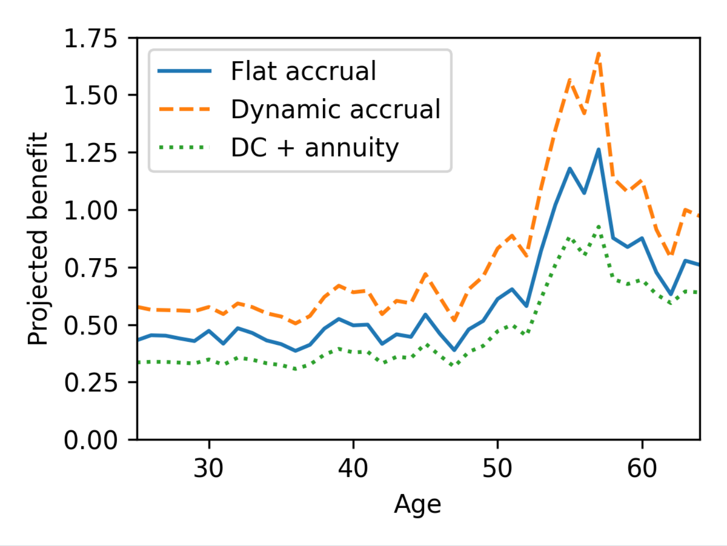
Stochastic cross subsidies for dynamic accrual

Left: cross-subsidies in dynamic-accrual CDC. Right: residuals of linear model on h, age, h*age.



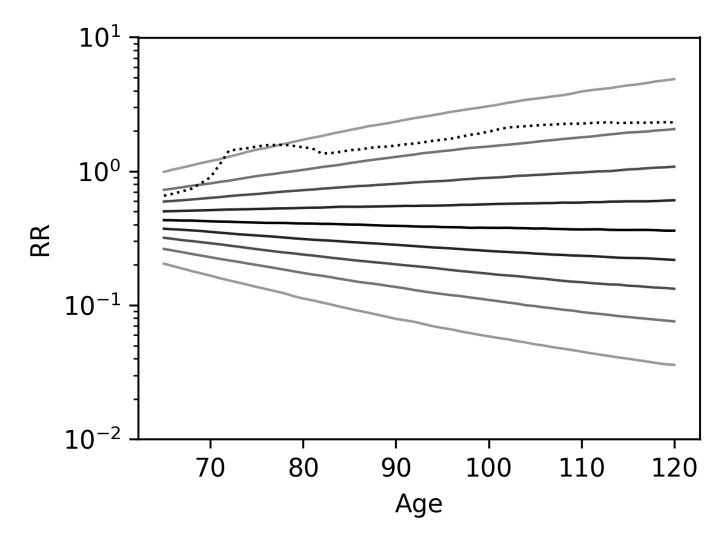
Projected benefit

Sensitivity to age $* h$ is ten times lower than predicted from standard discounting rule



Income for life?

Fan diagram of income in retirement for a dynamic-accrual scheme.



Alternative: Collective drawdown

- Record assets held by each individual at all times
- At each time period share the wealth left by the deceased among the survivors (a *tontine*)
- Asset selection and drawdown rules can be chosen as desired - e.g. trustees could select an "optimal strategy"

Alternative: Collective drawdown

- Record assets held by each individual at all times
- At each time period share the wealth left by the deceased among the survivors (a *tontine*)
- Asset selection and drawdown rules can be chosen as desired - e.g. trustees could select an “optimal strategy”

Simple tontine rules:

- Wealth is shared in proportion to

$$\text{probability of dying over period} \times \text{wealth at end of period}$$

- Separates investment risk from longevity-risk pooling

Alternative: Collective drawdown

- Record assets held by each individual at all times
- At each time period share the wealth left by the deceased among the survivors (a *tontine*)
- Asset selection and drawdown rules can be chosen as desired - e.g. trustees could select an "optimal strategy"

Simple tontine rules:

- Wealth is shared in proportion to

$$\text{probability of dying over period} \times \text{wealth at end of period}$$

- Separates investment risk from longevity-risk pooling

Communication:

- Member communications describe projected real-term benefits not pot size

No mutually beneficial contracts

Theorem

In a complete market there are no mutually beneficial contracts (subject to mild assumptions)

No mutually beneficial contracts

Theorem

In a complete market there are no mutually beneficial contracts (subject to mild assumptions)

Corollary

If the only risk is individual longevity risk, individual optimal investment with tontine is optimal as $N \to \infty$.

No mutually beneficial contracts

Theorem

In a complete market there are no mutually beneficial contracts (subject to mild assumptions)

Corollary

If the only risk is individual longevity risk, individual optimal investment with tontine is optimal as $N \to \infty$.

Essential points:

- Finiteness: Finite time horizon to avoid Ponzi scheme
- Non-saturation: Giving money away makes you less happy
- Non-compulsion: When you enter any contract you could choose to invest in the market instead. No central planner, no investing before you are born.
- Relevance: Preferences depends on market only

Infinite identical individuals

- Assume only individual longevity risk
- Assume there are an infinite number of individuals so tontine gives deterministic amounts to survivors
- Optimal insured drawdown is now a conventional optimal control problem

Example Loss Function:

Let c_j be consumption at time j. Let τ be time of death. Take

$$u(c, a) = \frac{c^\rho}{\rho} - \frac{a^\rho}{\rho}$$

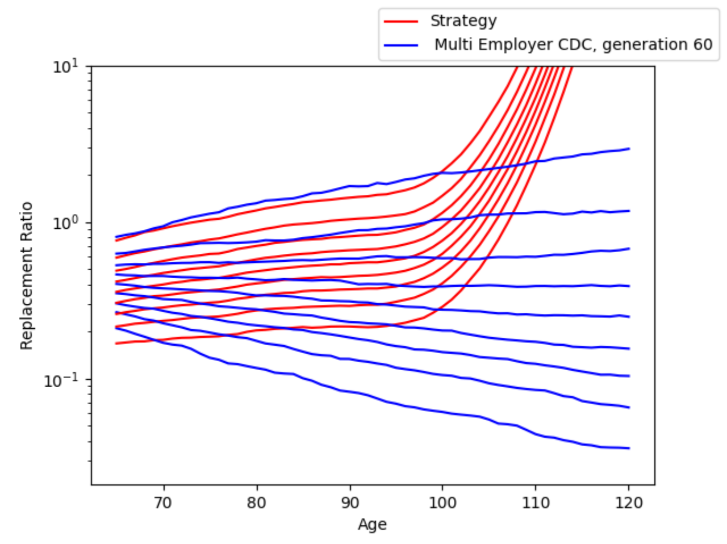
and

$$\text{loss} = \mathbb{E}\left(\exp\left(-\alpha \sum_{j=0}^{j<\tau} u(c_j, a)\right)\right).$$

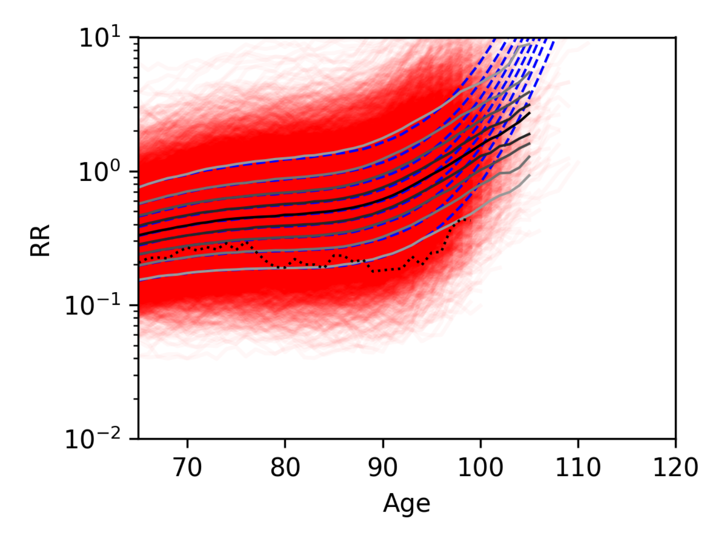
We solved this using a neural network in a Black-Scholes model parameteriized as for shared-indexation funds. [Link]

Neural Network solution vs shared-indexation

Parameters of utility chosen for ease of comparison to shared-indexation, assuming infinite identical individuals.



Finite fund size (20 per generation)



Conclusions

- Shared-indexation CDC is complex and does not appear to deliver the hoped-for benefits
- Collective-drawdown using a tontine is simple and gives significant benefits
- Other mutual insurance contracts may be beneficial, but at the expense of complexity

Thank you!